Revision Notes for FP1

Chapter 1 – Complex Numbers

$\sqrt{-1} = i$ or $i^2 = -1$. This means $\frac{1}{i} = -i$.

The conjugate of z = a + b*i* is **z* = a – b*i***.

The modulus of z = $|\boldsymbol{z}| = \sqrt{\boldsymbol{a^2 + b^2}}$. $|z_1 \times z_2| = |z_1| \times |z_2|$

The argument of z = **arg (z)** = $\boldsymbol{tan^{-1}\left(\frac{b}{a}\right)}$. This will give a value between –π and π, you may need to use common sense to change this to give the actual direction required.

Quadratics with real coefficients, solutions are either both real or **complex conjugates** (a + b*i* and a – b*i*)

Cubics with real coefficients have either 3 real roots, or 1 real root and complex conjugates.

Quartics with real coefficients have either 4 real roots, or 2 real roots and complex conjugates, or 2 complex conjugates.

Chapter 2 – Numerical Solutions

To prove a root exists between two limits, evaluate both limits and comment that the sign has changed.

To get a better approximation for a root, you can either:

(a) Interval Bisection. Find the midpoint, evaluate its value and use the sign to see which side the root now lies.

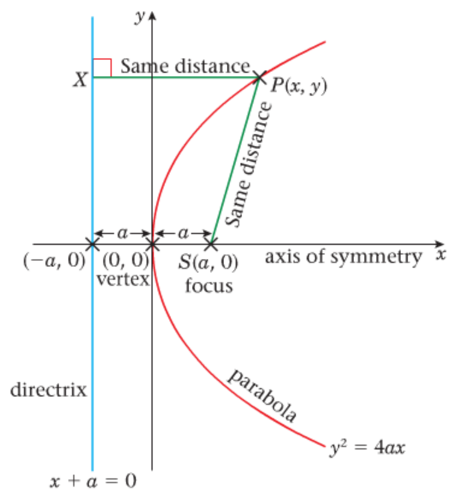
(b) Linear Interpolation. Draw a diagram, show relevant values and use ratios to get closer to root.

(c) Newton-Raphson. If trying to find $f(x) = 0$ then $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

Chapter 3 – Coordinate Systems

To change from parametric to Cartesian, eliminate the parameter and combine *x* and *y* into one equation.

	Parabola	Rectangular Hyperbola
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2 , 2at)$	$\left(ct , \frac{c}{t}\right)$
Foci	$(a , 0)$	This table is included in the formulae book.
Directrices	x = - a	



Chapter 4 – Matrix Algebra

$\begin{pmatrix} 5 & 2 & 3 \\ 1 & 0 & 4 \end{pmatrix}$ is a 2 x 3 matrix. The identity matrix is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

The determinant of a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $ad - bc$.

The inverse of a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Matrices can be used to represent transformations.

Reflection and rotation are included in the formulae book, look at where the signs are different to decide whether it is a rotation (top right / bottom left) or reflection (top left / bottom right).

AB means matrix B followed by matrix A.

Chapter 5 – Series

$$\sum_{r=1}^{n} r = \frac{n}{2}(n+1)\ , \qquad \sum_{r=1}^{n} 1 = n.$$

In the formulae book, we have the following:

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)\ , \qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2 = \left(\sum_{r=1}^{n} r\right)^2$$

Watch out for:

$$\sum_{r=k}^{n} u_r = \sum_{r=1}^{n} u_r - \sum_{r=1}^{k-1} u_r$$

Chapter 6 – Induction

Your solution must end with the following:

- (i) General statement is correct for n = 1.
- (ii) If statement is correct for n = k, then correct for n = k + 1
- (iii) Hence true for all n.

Trick: If proving divisible by 4, then writing 5^{n+1} as $4 \times 5^n + 5^n$ might help.

Remember to write down what you are trying to achieve to help you gain the solution.